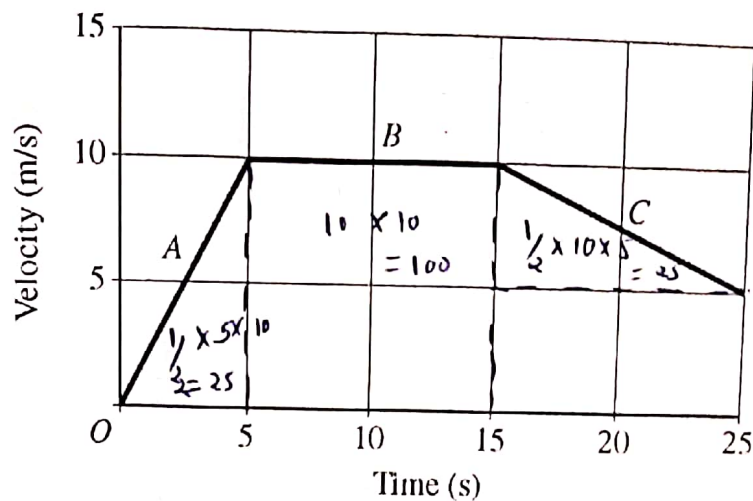
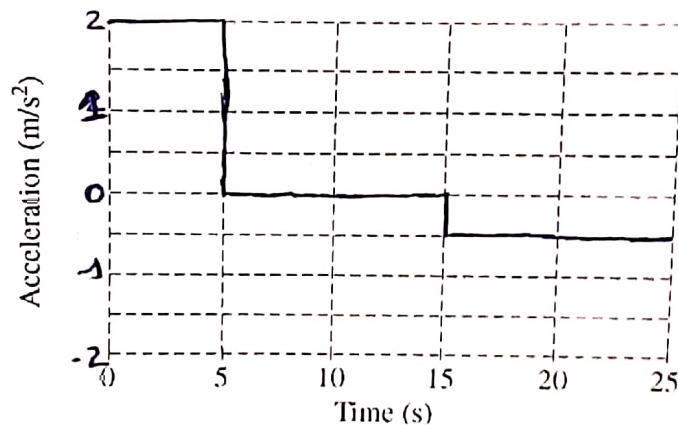


#1



A 0.40 kg object moves in a straight line under the action of a net force. The graph above shows the velocity as a function of time for the object during a 25 s interval. At time $t = 0$, the object is at the position $x = 0$.

(a) On the grid below, sketch a graph of the acceleration as a function of time for the object. Label the scale for the acceleration.



(b) Calculate the position of the object at $t = 5.0$ s.

$$\begin{aligned} \text{Position at } t = 5 \text{ s} &= \text{Area of } v-t \text{ graph} \\ &= \frac{1}{2} b \times h = \frac{1}{2} \times 10 \times 5 = 25 \text{ m} \\ &= \underline{\underline{25 \text{ m}}} \end{aligned}$$

(c) On which segment of the graph is the net force acting on the object zero?

 A ☒ B C

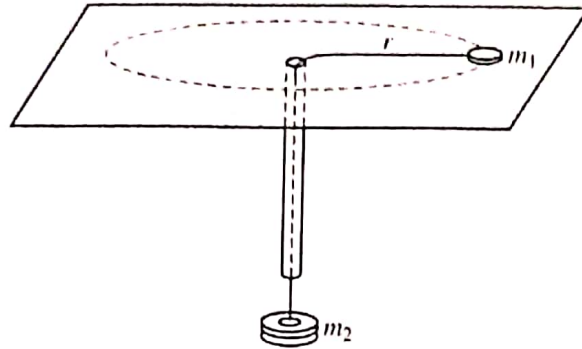
Justify your answer.

B $\rightarrow$ Where there is no acceleration means zero force.

(d) Calculate the net force on the object during the first 3.0 s of the motion.

$$\begin{aligned} F &= M \times a \\ F &= 0.4 \times 2 = \underline{\underline{0.8 \text{ N}}} \end{aligned}$$

#2



An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r , while another student measures the period P .

(a) Derive the equation $P = 2\pi \sqrt{\frac{m_1 r}{m_2 g}}$ that relates P and m_2 .

$$F_c = M_2 g = \frac{M_1 v^2}{r}$$

$$\frac{M_1 v^2}{r} = M_2 g$$

$$v = \frac{2\pi r}{P}$$

$$\frac{M_1 v^2}{r} = \frac{M_1 \times \left(\frac{2\pi r}{P}\right)^2}{r} = \frac{M_1 4\pi^2 r}{P^2}$$

$$\frac{M_1 4\pi^2 r}{P^2} = M_2 g$$

$$M_2 g P^2 = M_1 4\pi^2 r$$

$$P^2 = \frac{M_1 4\pi^2 r}{M_2 g}$$

$$P = \sqrt{\frac{4\pi^2 M_1 r}{M_2 g}} = 2\pi \sqrt{\frac{M_1 r}{M_2 g}}$$

The procedure is repeated, and the period P is determined for four different values of m_2 , where $m_1 = 0.012 \text{ kg}$ and $r = 0.80 \text{ m}$. The data, which are presented below, can be used to compute an experimental value for g .

$m_2 \text{ (kg)}$	0.020	0.040	0.060	0.080
$P \text{ (s)}$	1.40	1.05	0.80	0.75
$1/P^2$	0.51	0.91	1.6	1.8

(b) What quantities should be graphed to yield a straight line with a slope that could be used to determine g ?

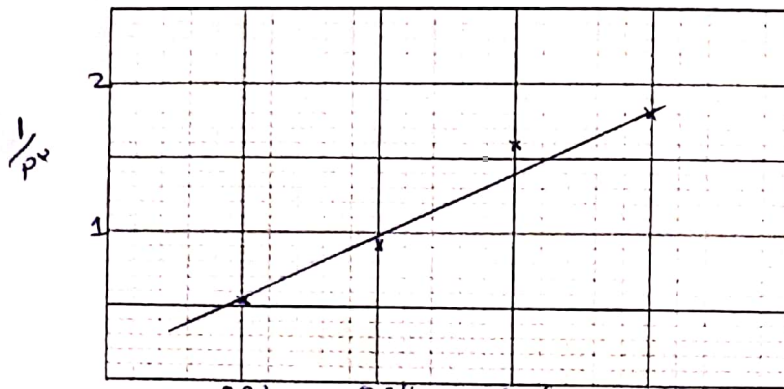
$$P = 2\pi \sqrt{\frac{M_1 r}{M_2 g}} \text{ so quantities; } P^2 \text{ and } \frac{1}{M_2}$$

$$P = a \sqrt{\frac{1}{M_2}}$$

$$\text{or } \frac{1}{P^2} \text{ and } M_2$$

$$\text{or } P \text{ and } \frac{1}{\sqrt{M_2}}$$

(c) On the grid below, plot the quantities determined in part (b), label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.



$$\frac{1}{p^2} = \frac{M_2 g}{4\pi^2 M_1 r}$$

$$x\text{-axis} = M_2(1/g)$$

$$y\text{-axis} = \frac{1}{p^2}$$

(d) Use your graph to calculate the experimental value of g

$$\frac{1}{p^2} = \frac{M_2 g}{4\pi^2 M_1 r}$$

$$\text{slope} = \frac{g}{4\pi^2 M_1 r}$$

$$g \Rightarrow \text{slope} \times 4\pi^2 M_1 r$$

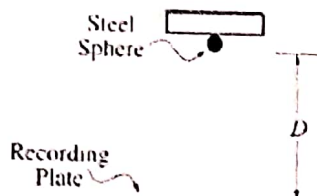
$$\text{slope} = \frac{1.8 - 0.51}{0.08 - 0.02} = 21.5$$

$$g = 21.5 \times 0.012 \text{ kg} \times 0.08 \text{ m} \times 4\pi^2$$

$$= 8.15$$

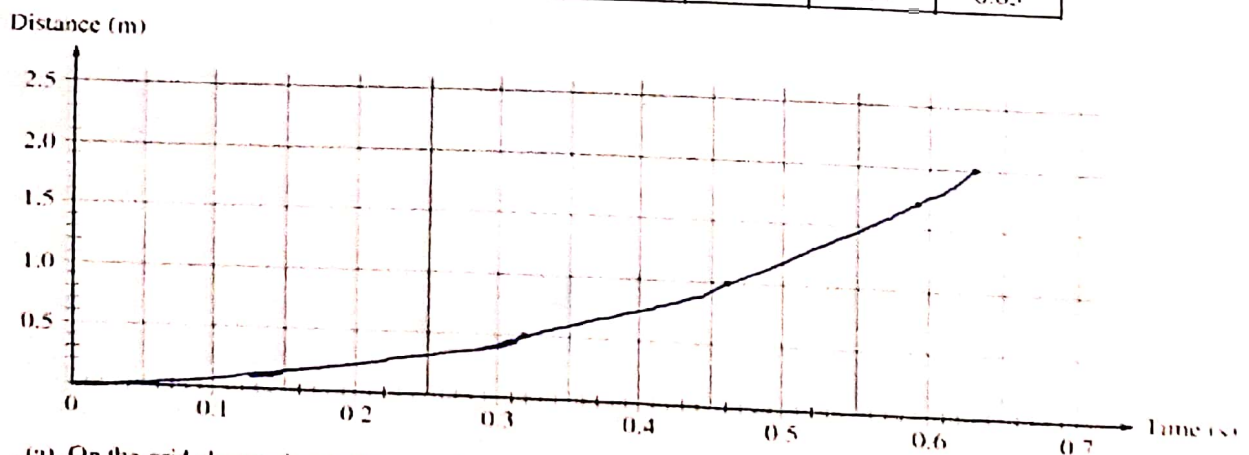
$$g = 8.15 \text{ m/s}^2$$

#3



A student wishing to determine experimentally the acceleration g due to gravity has an apparatus that holds a small steel sphere above a recording plate, as shown above. When the sphere is released, a timer automatically begins recording the time of fall. The timer automatically stops when the sphere strikes the recording plate. The student measures the time of fall for different values of the distance D shown above and records the data in the table below. These data points are also plotted on the graph.

Distance of Fall (m)	0.10	0.50	1.00	1.70	2.00
Time of Fall (s)	0.14	0.32	0.46	0.59	0.63



(a) On the grid above, sketch the smooth curve that best represents the student's data

The student can use these data for distance D and time t to produce a second graph from which the acceleration g due to gravity can be determined.

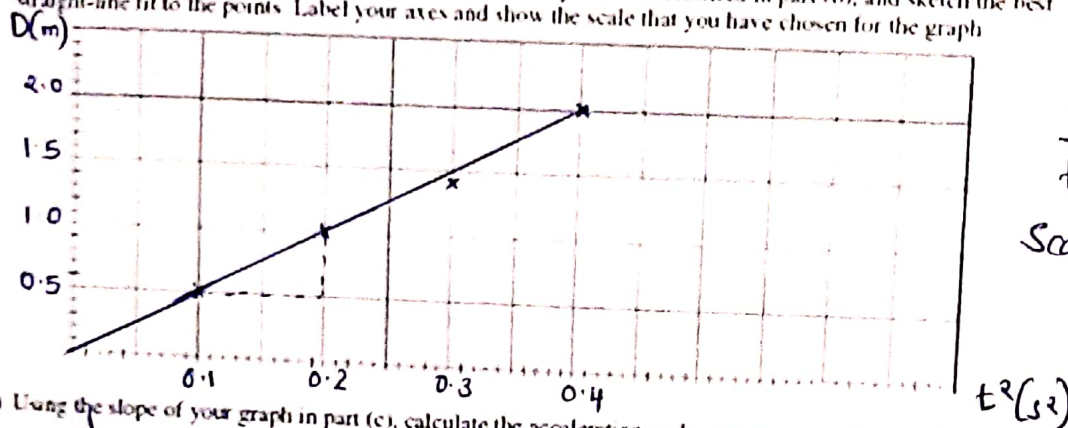
(b) If only the variables D and t are used, what quantities should the student graph in order to produce a linear relationship between the two quantities?

$$D = \frac{1}{2}gt^2$$

$$D \text{ and } t^2$$

To yield straight line quantities to graph are: $\sqrt{D}$ or t

(c) On the grid below, plot the data points for the quantities you have identified in part (b), and sketch the best straight-line fit to the points. Label your axes and show the scale that you have chosen for the graph.



D	0.5	1.0	1.5	2.0
t^2	0.1	0.2	0.3	0.4

Scale: y-axis
1 cm = 0.5 m
x-axis
1 cm = 0.05 s²

(d) Using the slope of your graph in part (c), calculate the acceleration g due to gravity in this experiment.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{1.0 - 0.5}{0.2 - 0.1} = 5 \text{ m/s}^2$$

$$\text{slope of the line shown} = 5 \text{ m/s}^2 \text{ giving } g = 10 \text{ m/s}^2$$

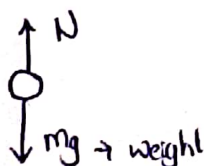
(e) State one way in which the student could improve the accuracy of the results if the experiment were to be performed again. Explain why this would improve the accuracy.

Doing several trials for each value of given D and then take the mean values of the trials. This helps reduce random errors which improves accuracy.

#4

A student of mass m stands on a platform scale in an elevator in a tall building. The positive direction for all vector quantities is upward.

(a) Draw a free-body diagram showing and labeling all the forces acting on the student, who is represented by the dot below.



(b) Derive an expression for the reading on the scale in terms of the acceleration a of the elevator, the mass m of the student, and fundamental constants.

$$F_{\text{net}} = ma$$

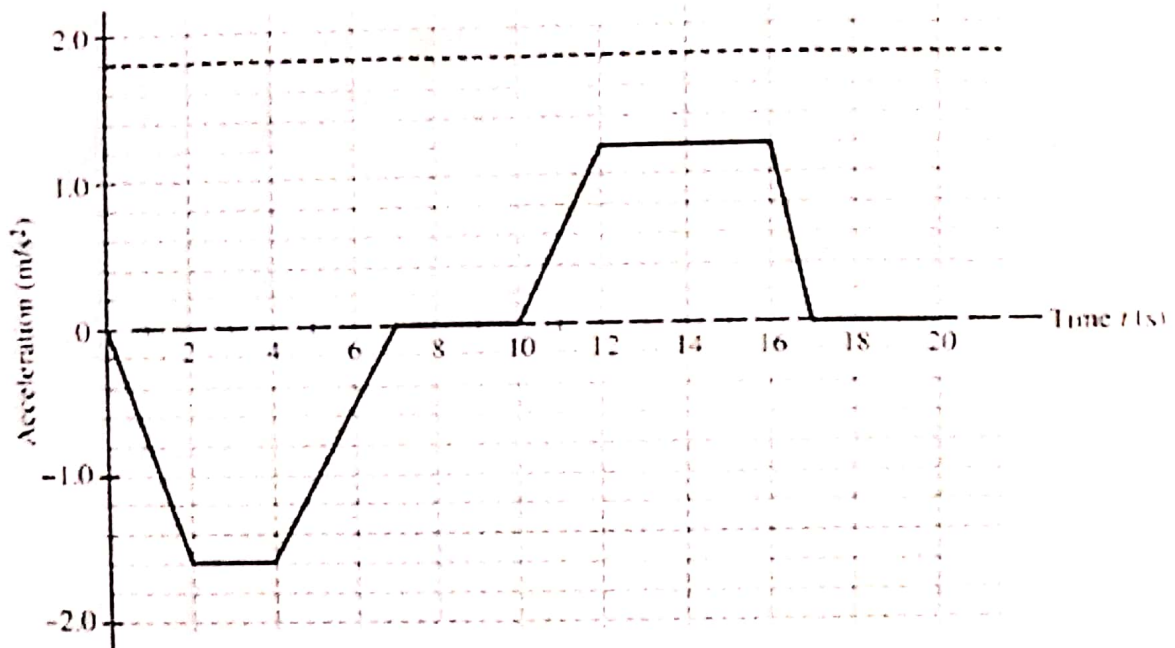
$$F_{\text{net}} = N - mg$$

$$N - mg = ma$$

$$N = mg + ma$$

$$N = m(g + a)$$

An inspector provides the student with the following graph showing the acceleration a of the elevator as a function of time t



(c)

- i. During what time interval(s) is the force exerted by the platform scale on the student a maximum value?

interval beginning from 12 seconds and ending at 18 seconds

- ii. Calculate the magnitude of that maximum force for a 45 kg student.

$$N = m(g + a)$$

$$N = 45(9.8 \text{ m/s}^2 + 1.2 \text{ m/s}^2)$$

$$N = 45 \times 11.0 \text{ m/s}^2 = 495 \text{ N}$$

$$N_{\text{max}} = 500 \text{ N}$$

$$N_{\text{max}} = 500 \text{ N}$$

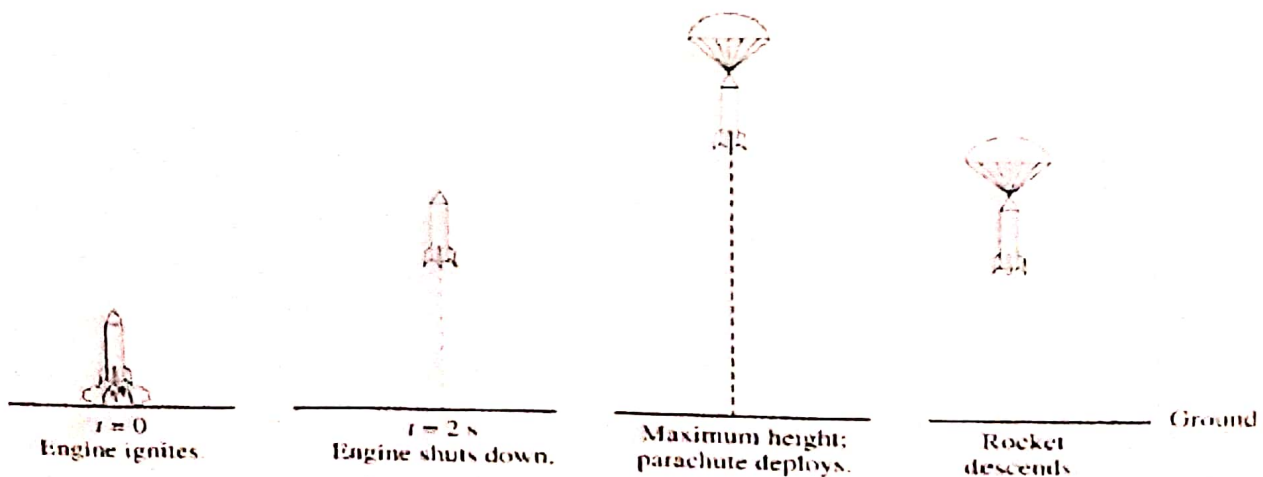
- (d) During what time interval(s) is the speed of the elevator constant?

From 6.5 sec to 10 sec

and

From 16.5 sec to 20 sec

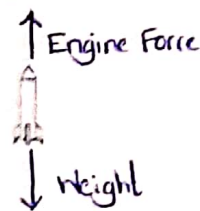
#5



A model rocket of mass 0.250 kg is launched vertically with an engine that is ignited at time $t = 0$, as shown above. The engine provides an impulse of $20.0 \text{ N}\cdot\text{s}$ by firing for 2.0 s . Upon reaching its maximum height, the rocket deploys a parachute, and then descends vertically to the ground.

(a) On the figures below, draw and label a free-body diagram for the rocket during each of the following intervals.

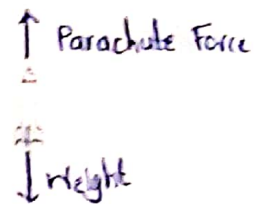
i. While the engine is firing



ii. After the engine stops, but before the parachute is deployed



iii. After the parachute is deployed



(b) Determine the magnitude of the average acceleration of the rocket during the 2 s firing of the engine.

$$J_{\text{engine}} = F_{\text{eng}} t$$

$$20 = F_{\text{eng}} (2) \quad F_{\text{eng}} = 10 \text{ N}$$

$$F_{\text{net}} = ma; (F_{\text{eng}} - mg) = ma$$

$$\therefore (10 - 0.25 \times 9.8) = 0.25a$$

$$7.55 = 0.25a$$

$$a = \underline{\underline{30.2 \text{ m/s}^2}}$$

(c) What maximum height will the rocket reach?

$$d_1 = v_i t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} \times 30 \times 2^2 = 60 \text{ m}$$

distance for free fall

$$v_{1f} = v_{1i} + a_1 t_1 = 0 + 30 \times 2 = 60 \text{ m/s}$$

$$v_{2f}^2 = v_{2i}^2 + 2g d_2 = 60^2 + 2 \times (-9.8) (d_2)$$

$$0 = 60^2 + (-19.6 d_2)$$

$$3600 = 19.6 d_2$$

$$d_2 = 184 \text{ m}$$

$$\text{total distance} = 184 + 60 = \underline{\underline{244 \text{ m}}}$$

(d) At what time after $t = 0$ will the maximum height be reached?

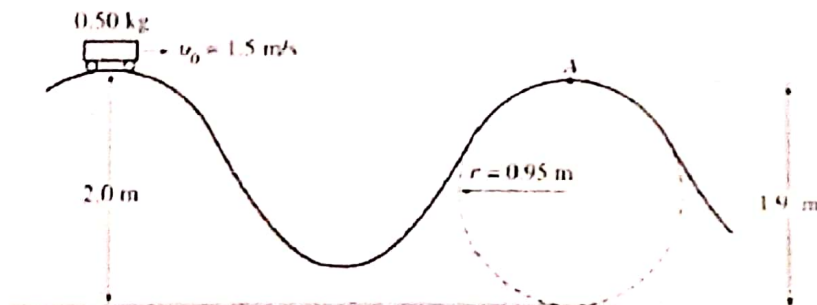
$$v_{2f} = v_{2i} + g t$$

$$0 = 60 + (-9.8 t)$$

$$9.8 t = 60$$

$$t = \underline{\underline{6.1 \text{ s}}}$$

#6



1. (15 points)

A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed $v_0 = 1.5 \text{ m/s}$ at the top of the first hill of height 2.0 m . Point A is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m .

(a) Calculate the speed of the car at point A.

Energy Conservation:

$$\frac{1}{2} M v_0^2 + M g h_0 = \frac{1}{2} M v_A^2 + M g h_A$$

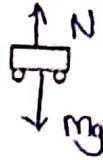
$$v_A^2 = v_0^2 + 2g(h_0 - h_A)$$

$$v_A^2 = (1.5 \text{ m/s})^2 + 2 \times (9.8 \text{ m/s}^2) (2 \text{ m} - 1.9 \text{ m}) = 4.21 (\text{m/s})^2$$

$$v_A = \sqrt{4.21 (\text{m/s})^2} = 2.05$$

$$v_A = \underline{\underline{2.05 \text{ m/s}}}$$

- (b) On the figure of the car below, draw and label vectors to represent the forces on the car at point A



- (c) Calculate the magnitude of the force of the track on the car at point A

The net force = Centripetal force at point A

$$F_{\text{net}} = Mg - N = \frac{MV_A^2}{r}$$

$$N = Mg - \frac{MV_A^2}{r} = (0.5 \text{ kg} \times 9.8 \text{ m/s}^2) - \frac{(0.5 \times 4.21^2 \text{ m}^2/\text{s}^2)}{0.95 \text{ m}} = \underline{\underline{2.7 \text{ N}}}$$

- (d) In order to stop the car at point A, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point A.

The work done = the kinetic energy at point A in absence of friction

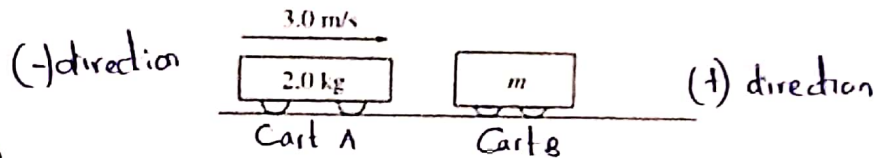
$$W_{\text{friction}} = -\frac{1}{2} MV_A^2$$

$$= -\frac{1}{2} \times 0.5 \times 4.21^2 = -1.05 = \underline{\underline{-1.05 \text{ Joules}}}$$

- (e) Explain how to modify the track design to cause the car to lose contact with the track at point A before descending down the track. Justify your answer.

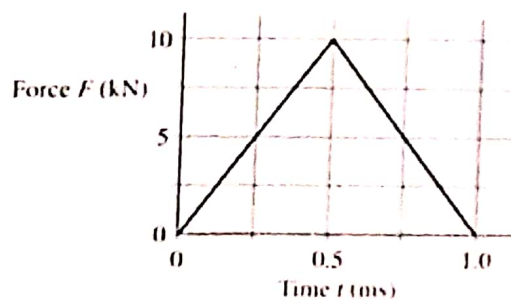
→ The larger the quantity of $\frac{MV^2}{r}$, the more likely the car is to lose contact with track.
 Since more centripetal force would be required to keep it there.
 → To increase centripetal force, you either increase velocity or reduce radius.
 → To increase velocity of car, initial hill should be higher to increase initial energy.
 → To reduce radius, shorten the length of the hill.

(#7)



(15 points)

A 2.0 kg frictionless cart is moving at a constant speed of 3.0 m/s to the right on a horizontal surface, as shown above, when it collides with a second cart of undetermined mass m that is initially at rest. The force F of the collision as a function of time t is shown in the graph below, where $t = 0$ is the instant of initial contact. As a result of the collision, the second cart acquires a speed of 1.6 m/s to the right. Assume that friction is negligible before, during, and after the collision.



$$F \Rightarrow \text{kN} \\ 1 \text{ kN} = 10^3 \text{ N}$$

$$\text{time} = \text{ms} \\ 1 \text{ s} = 1000 \text{ ms}$$

- (a) Calculate the magnitude and direction of the velocity of the 2.0 kg cart after the collision

$$\begin{aligned} \Delta p &= \text{area under the curve} \\ &= \frac{1}{2} b \times h = \frac{1}{2} \times 1 \times 10^{-3} \text{ s} \times 10 \times 10^3 \text{ N} \\ &= 5 \text{ N} \cdot \text{s} = 5 \text{ kg} \cdot \text{m/s} \end{aligned}$$

negative area
force pushes car to
left

$$\begin{aligned} \Delta p &= m \Delta v \\ -5 &= 2(V_f - 3) \\ -\frac{5}{2} &= V_f - 3 \\ -V_f &= -3 + \frac{5}{2} \end{aligned}$$

$$\begin{aligned} V_f &= 3 - \frac{5}{2} \\ V_f &= 0.5 \text{ m/s to the right direction} \end{aligned}$$

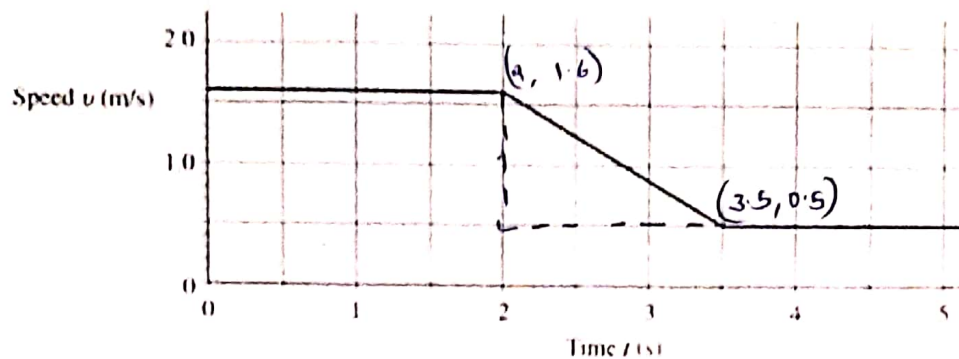
(b) Calculate the mass m of the second cart

$P = \frac{MV}{\text{Cart A}}$

	Cart A	Cart B	Total
Before	2×3	0	6
After	2×0.5	$m \times 1.6$	6

$2 \times 0.5 + m \times 1.6 = 6$
 $m \times 1.6 = 5$
 $m = \frac{5}{1.6} = 3.125 \text{ kg}$
 $= \underline{3.125 \text{ kg}}$

After the collision, the second cart eventually experiences a ramp, which it traverses with no frictional losses. The graph below shows the speed v of the second cart as a function of time t for the next 5.0 s, where $t = 0$ is now the instant at which the carts separate.



(c) Calculate the acceleration of the cart at $t = 3.0$ s.

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0.5 - 1.6}{3.5 - 2} = \frac{-1.1}{1.5} = -0.73$

Acceleration = -0.73 m/s^2

at 3s, it is decelerating $a = \underline{-0.73 \text{ m/s}^2}$

(d) Calculate the distance traveled by the second cart during the 5.0 s interval after the collision ($0 \leq t \leq 5.0$ s)

distance = total area under the curve

$A_1 = 2 \times 1.6 = 3.2$

$A_2 = \frac{1}{2} \times 1.5 \times 1.1 = 0.825$

$A_3 = 3 \times 0.5 = 1.5$

$\text{Total} = A_1 + A_2 + A_3$
 $= 3.2 + 0.825 + 1.5$
 $= \underline{5.525 \text{ m}}$

(e) State whether the ramp goes up or down and calculate the maximum elevation (above or below the initial height) reached by the second cart on the ramp during the 5.0 s interval after the collision ($0 \leq t \leq 5.0$ s)

(i) The ramp goes up $\Rightarrow$ There is a decrease in speed.

(ii) By conservation of energy

	PE_g	PE_e	KE	Total
Before	0	0	4	4
After	30.625	0	0.39	4

$PE_g = Mgh$
 $= 3.125 \times 9.8 \times h$
 $= 30.625 h$

Before $KE = \frac{1}{2} \times m v^2$

$= \frac{1}{2} \times 3.125 \times 1.6^2$

$= 4 \text{ Joules} \rightarrow \text{before}$

After $= \frac{1}{2} \times 3.125 \times 0.5^2$

$= 0.39 \text{ Joules} \Rightarrow \text{After}$

$\text{Therefore } 30.625 h + 0.39 = 4$
 $30.625 h = 3.61$
 $h = \underline{0.12 \text{ m}}$